STATEMENT OF BOUNDARY AND CONJUGATION CONDITIONS FOR PROBLEMS OF HEAT TRANSFER IN GRANULAR BEDS ON THE BASIS OF A TWO-TEMPERATURE MODEL

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Physically justified boundary and conjugation conditions for problems of heat transfer in infiltrated granular beds have been formulated within the framework of the two-temperature model that takes into account the absence of interphase interaction at the boundaries. It is shown that the classical Danckwerts conditions are applicable to a gas. The problem of filtration cooling of a heat-generating granular bed over which there is an inert bed (pile-up) has been solved in a new statement. The dependence of the pressure drop in a granular bed on the mass flow rate of the gas is established. A formula to calculate the maximum temperature of particles is obtained. The region of applicability of the one-temperature model is determined.

In engineering, there are a large number of thermal processes that proceed in fixed blown-through granular beds, where temperature drops between particles and gas are to be taken into account, thus rejecting the representation of the bed as a homogeneous heat-conducting medium. In the first place, these are various unsteady-state processes of heating or cooling of the bed by a heat-carrier flow, when the temperatures of the phases have insufficient time to equilibrate. Another example concerns the case of the occurrence of temperature drops in moving of a heat flux opposite to the gas (transpiration cooling of the surfaces of flying vehicles, blades of high-temperature gas turbines, etc.). Finally, mention should be made of the processes of filtration cooling of a heat-generating granular bed that can be encountered in nuclear power plants (cooling of fuel microelements) under both standard and emergency conditions.

In order to describe the hydrodynamics and heat transfer inside a heat-generating granular bed under steady-state conditions, the following system of equations is used:

$$\rho_f \varepsilon v_f \frac{dv_f}{dx} = -\frac{dp}{dx} - 150 \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu_f u_f}{d^2} - 1.75 \cdot \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho_f u_f^2}{d} , \quad (1)$$

$$c_p \rho_f \varepsilon v_f \frac{dT_f}{dx} = \frac{d}{dx}\left(\varepsilon \lambda_f \frac{dT_f}{dx}\right) + \frac{6(1-\varepsilon)\alpha}{d}(T_s - T_f) , \quad (2)$$

$$0 = \frac{d}{dx}\left((1-\varepsilon)\lambda_s \frac{dT_s}{dx}\right) + \frac{6(1-\varepsilon)\alpha}{d}(T_f - T_s) + Q(1-\varepsilon) , \quad (3)$$

$$p = \rho_f R T_f . \quad (4)$$

The force of gas friction against particles is presented in (1) on the basis of the well-known Ergun equation [1]. The gas is assumed to be perfect. The effective heat-conduction coefficients of the phases λ_f and λ_s are defined as [2]

$$\frac{\lambda_f}{\lambda_f^0(T_f)} = 1 + \frac{\lambda_e}{\lambda_f^0(T_f)} , \quad (5)$$

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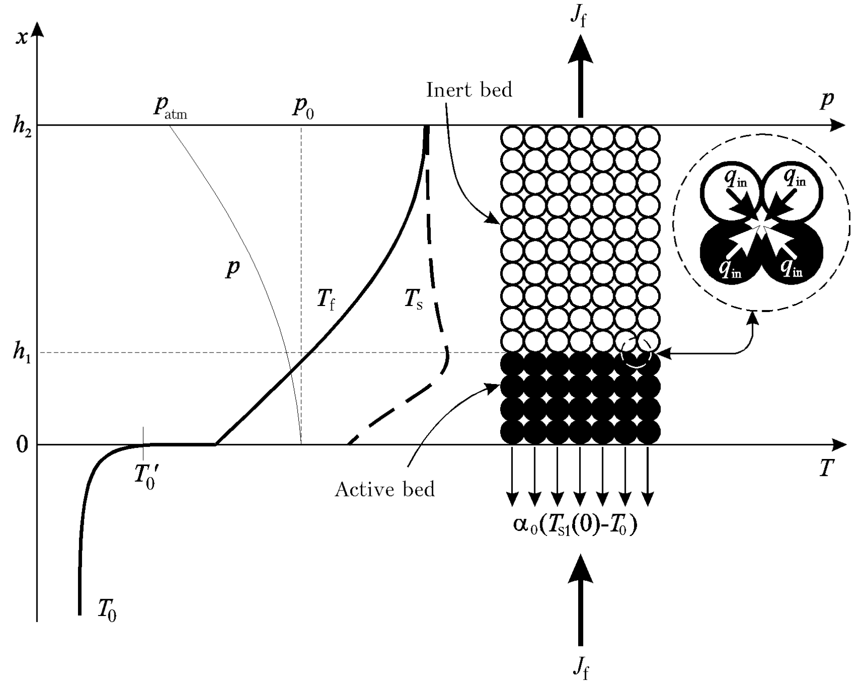

Fig. 1. System of coordinates, directions of mass and heat fluxes, and the character of distribution of the temperatures of phases and pressure inside the granular bed.

where

$$\frac{\lambda_e}{\lambda_f^0(T_f)} = 0.03\,\mathrm{Re}\,(T_f)\,\mathrm{Pr}\,(T_f)\,. \tag{6}$$

The thermal conductivity of an ensemble of particles is

$$\lambda_s = \lambda_{c-c} + \lambda_r\,, \tag{7}$$

where the thermal conductivity of dead zones λ_{c-c} is

$$\frac{\lambda_{c-c}}{\lambda_f^0(T_s)} = 12 + 0.85\,\mathrm{Re}\,(T_s)\,\mathrm{Pr}\,(T_s)\,, \tag{8}$$

and the radiant component λ_r [3, 4] is

$$\lambda_r = \frac{0.3024}{\kappa + \sigma}\left(\frac{T_s}{100}\right)^3 . \tag{9}$$

The coefficient of interphase heat transfer is determined by the formulas [5]

$$\mathrm{Nu} = \frac{\alpha d}{\lambda_f^0} = 0.4\left(\frac{\mathrm{Re}}{\varepsilon}\right)^{2/3}\mathrm{Pr}^{1/3}\,,\quad \frac{\mathrm{Re}}{\varepsilon} > 200\,, \tag{10}$$

$$\mathrm{Nu} = \frac{\alpha d}{\lambda_f^0} = 1.6\cdot 10^{-2}\left(\frac{\mathrm{Re}}{\varepsilon}\right)^{1.3}\mathrm{Pr}^{1/3}\,,\quad \frac{\mathrm{Re}}{\varepsilon} \le 200 \;\text{ at }\; T = \frac{T_f + T_s}{2}\,. \tag{11}$$

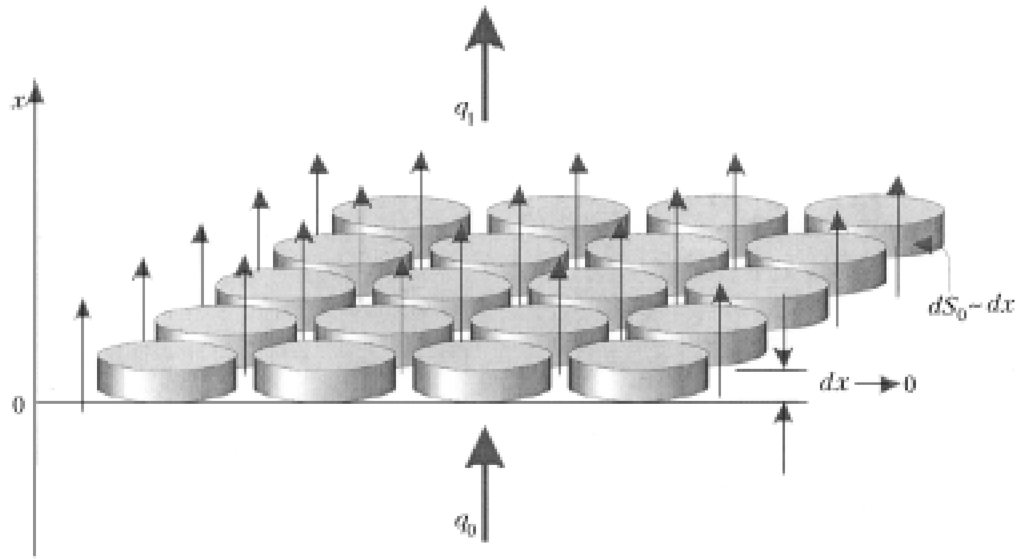

Fig. 2. Toward the derivation of the boundary condition for the gas at $x = 0$.

Since the quantities ρ_f, μ_f, and λ_f^0 depend on temperature and pressure, we have approximated data on the thermophysical properties of air in the range of temperatures from 300 to 1400 K and pressures from 1 to 20 atm [6]:

$$\rho_f = 0.00352 p / T_f \,, \quad \mu_f = 2.64 \cdot 10^{-7} T^{0.74} \,, \quad \lambda_f^0 = 0.00021 T^{0.84} \,. \tag{12}$$

An analysis of examples that demonstrate the application of system (1)–(4) to modeling concrete processes [2, 7–10] has shown that the main unsolved problem is the formulation of physically justified boundary conditions and conjugation conditions, similar to the boundary conditions of the fourth kind that appear at the interface between disperse media having different characteristics.

We will consider the process of heat transfer when a gas of temperature T_0 at a pressure p_0 enters a dispersed bed of height h_1, in which heat of constant power Q is released. This bed will be called active. Above this bed, there is another bed of height h_2–h_1, where heat release is absent (an inert bed or "pile-up" [7]). The active and inert beds are characterized by the quantities ε_1 and d_1 and ε_2 and d_2, respectively. In the process of filtration, the gas moves through the beds, gets heated as a result of heat transfer, and outflows into a free space at a pressure p_{atm} (Fig. 1). We will formulate the boundary and conjugation conditions at the active bed–inert bed interface.

Boundary Conditions. *1. Boundary condition at $x = 0$ (gas).* We will consider an elementary volume of the active bed $dV = Sdx$ near the interface $x = 0$ (Fig. 2). The balance of the gas heat fluxes is

$$(q_1 - q_0)\, S = \alpha_1 \,(T_{s1} - T_{f1})\, dS_{in} \,. \tag{13}$$

With allowance for $dS_{in} = S_{in} dV = S_{in} S dx$, we obtain

$$q_1 - q_0 = \alpha_1 \,(T_{s1} - T_{f1})\, S_{in} dx \,. \tag{14}$$

When $dx \to 0$, Eq. (14) yields the unknown boundary condition

$$q_1 = q_0 \,. \tag{15}$$

Since $q_0 = c_p \rho_{f0} u_{f0} T_0'$ and $q_1 = c_p \rho_{f1} u_{f1} T_{f1}'(0) - \varepsilon_1 \lambda_{f1} \left.\dfrac{dT_{f1}}{dx}\right|_{x=0}$, boundary condition (15) at $x = 0$ takes the form

$$c_p \frac{J_f}{\varepsilon_1} (T_{f1} - T_0') = \lambda_{f1} \frac{dT_{f1}}{dx} \,. \tag{16}$$

Equation (16) is the well-known Danckwerts condition [11] widely used in modeling the processes of transfer in granular beds which are considered as homogeneous media. In this connection, the following conclusion can be drawn, which is important in the context of the present work: in formulating boundary conditions for the two-temperature model, the phases at the boundaries of the bed can be considered as isolated from each other because of the absence of the interface surface there.

2. Boundary condition at $x = 0$ (particles). With allowance for the transfer of the heat that enters the system together with the gas (preliminary heating) (Fig. 1), the boundary condition at $x = 0$ has the form

$$\lambda_{s1}(1-\varepsilon_1)\frac{dT_{s1}}{dx} = \alpha_0 (T_{s1} - T_0) . \quad (17)$$

Using the relation $\lambda_{s1}(1-\varepsilon_1)\left.\frac{dT_{s1}}{dx}\right|_{x=0} = c_p J_f (T_0' - T_0)$, we rearrange Eq. (16) as

$$c_p J_f (T_{f1} - T_0) = \varepsilon_1 \lambda_{f1} \frac{dT_{f1}}{dx} + (1-\varepsilon_1)\, \lambda_{s1} \frac{dT_{s1}}{dx} . \quad (18)$$

3. Boundary condition at $x = h_2$ (gas). With allowance for the independence of the phases, we may use the second Danckwerts condition

$$\frac{dT_{f2}}{dx} = 0 , \quad (19)$$

which states that the entire heat flux transferred by the gas is equal to the convective one. We note that the hypothesis

$$T_{s2} = T_{f2} , \quad (20)$$

adopted in [9] disagrees with (19).

4. Boundary condition at $x = h_2$ (particles). At the absence of a heat flux from the outside, we have

$$\frac{dT_{s2}}{dx} = 0 . \quad (21)$$

In [7], Eq. (21) is replaced by the equation

$$\lambda_{s2}\frac{dT_{s2}}{dx} = \alpha_2 (T_{f2} - T_{s2}) , \quad (22)$$

which, as the foregoing analysis has shown, seems to be erroneous.

Conjugation Conditions ($x = h_1$). *1. Conjugation condition for pressure.* To obtain the sought-after condition, we use Eq. (1). With allowance for $J_f = \varepsilon\rho_f v_f = \text{const}$, we have

$$d(J_f v_f + p) = -\left(150\cdot\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu_f u_f}{d^2} + 1.75\cdot\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\rho_f u_f^2}{d}\right)dx . \quad (23)$$

We will integrate Eq. (23) within the limits $h_1 - \Delta x$, $h_1 + \Delta x$:

$$\int_{h_1-\Delta x}^{h_1+\Delta x} d(J_f v_f + p) = -\int_{h_1-\Delta x}^{h_1+\Delta x}\left(150\cdot\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu_f u_f}{d^2} + 1.75\cdot\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\rho_f u_f^2}{d}\right)dx . \quad (24)$$

When $\Delta x \to 0$, Eq. (24) yields the conjugation condition for pressure:

$$\Delta p = p_2 - p_1 = J_f (v_{f1} - v_{f2}) . \quad (25)$$

We note that when $\Delta x \to 0$, the integral on the right-hand side of Eq. (24) also tends to zero, since it contains the expression for the interface surfaces $\frac{\Delta S_i}{S} = \frac{6\,(1-\varepsilon_i)}{d_i}\Delta x$. In [10], for Δp a somewhat different formula was suggested:

$$\Delta p = \frac{J_{f1}}{\varepsilon_1}(v_{f1} - v_{f2}) , \tag{26}$$

which does not satisfy the needed symmetry condition.

2. Conjugation conditions for the gas temperatures and their derivatives. We will use Eq. (2) representing the gas enthalpy as $I_f = c_p T_f = c_v T_f + \frac{p}{\rho_f}$:

$$d\left(J_f\left(c_v T_f + \frac{p}{\rho_f}\right) - \varepsilon \lambda_f \frac{dT_f}{dx}\right) = \frac{6\,(1-\varepsilon)\,\alpha}{d}(T_s - T_f)\,dx . \tag{27}$$

Having performed, as before, the integration of Eq. (27), for $\Delta x \to 0$ we obtain

$$J_f c_v (T_{f1} - T_{f2}) + J_f\left(\frac{p_1}{\rho_{f1}} - \frac{p_2}{\rho_{f2}}\right) = \varepsilon_1 \lambda_{f1}\frac{dT_{f1}}{dx} - \varepsilon_2 \lambda_{f2}\frac{dT_{f2}}{dx} . \tag{28}$$

As is seen, relation (28) represents the conservation condition for the total heat flux of the gas, which can be written in the form

$$J_f c_p (T_{f1} - T_{f2}) = \varepsilon_1 \lambda_{f1}\frac{dT_{f1}}{dx} - \varepsilon_2 \lambda_{f2}\frac{dT_{f2}}{dx} . \tag{29}$$

Subject to $c_p - c_v = R$, a comparison of Eqs. (28) and (29) yields the sought-after condition for the temperature jump:

$$\Delta T_f = T_{f1} - T_{f2} = \left(\frac{p_2}{\rho_{f2}} - \frac{p_1}{\rho_{f1}}\right)\frac{1}{R} . \tag{30}$$

We note that condition (30) is also determined from the equation of state of the ideal gas (4). Substituting ΔT_f into Eq. (29), we obtain the condition for the jump of the derivatives:

$$\varepsilon_2 \lambda_{f2}\frac{dT_{f2}}{dx} - \varepsilon_1 \lambda_{f1}\frac{dT_{f1}}{dx} = J_f c_p \Delta T_f = \frac{J_f c_p}{R}\left(\frac{p_2}{\rho_{f2}} - \frac{p_1}{\rho_{f1}}\right). \tag{31}$$

In [10], somewhat different, more complex, expressions were obtained instead of Eqs. (30) and (31), which do not satisfy the symmetry condition.

3. Conjugation conditions for the temperatures of particles and their derivatives. It is evident that these are the conventional boundary conditions of the IVth kind:

$$T_{s1} = T_{s2} , \quad (1-\varepsilon_1)\,\lambda_{s1}\frac{dT_{s1}}{dx} = (1-\varepsilon_2)\,\lambda_{s2}\frac{dT_{s2}}{dx} . \tag{32}$$

We will write system (1)–(4) with boundary conditions (16), (17), (19), and (21) and conjugation conditions (25), (30)–(32) in dimensionless form:

$$\hat{J}_i \rho'_{fi} \frac{d}{d\xi}\left(\frac{1}{\rho'_{fi}}\right) = -D_i \frac{dp'_i}{d\xi} - 150 \cdot \frac{(1-\varepsilon_i)^2}{\varepsilon_i^3} \mathrm{Re}_i - 1.75 \cdot \frac{(1-\varepsilon_i)}{\varepsilon_i^3} \mathrm{Re}_i^2 , \tag{33}$$

$$\frac{d\theta_{fi}}{d\xi} = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_{fi}} \frac{d\theta_{fi}}{d\xi}\right) + \frac{1}{\mathrm{Pe}_i} (\theta_{si} - \theta_{fi}) , \tag{34}$$

$$0 = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_{si}} \frac{d\theta_{si}}{d\xi}\right) + \frac{1}{\mathrm{Pe}_i} (\theta_{fi} - \theta_{si}) + \hat{Q}\delta_{1i} , \tag{35}$$

$$\rho'_{fi} = \left(p'_i\left(1 - \frac{p_{atm}}{p_0}\right) + \frac{p_{atm}}{p_0}\right) \Big/ (\theta_{fi} + 1) \tag{36}$$

(i = 1, the active bed with $0 \le \xi < \xi_1$; i = 2, the inert bed with $\xi_1 \le \xi \le 1$).

The boundary conditions are

$$p'_1(0) = 1 , \quad p'_2(1) = 0 ; \tag{37}$$

$$\xi = 0 : \quad \theta_{f1} = \frac{1}{\mathrm{Pe}_{f1}} \frac{d\theta_{f1}}{d\xi} + \frac{1}{\mathrm{Pe}_{s1}} \frac{d\theta_{s1}}{d\xi} , \quad \theta_{s1} = 6\,(1-\varepsilon_1) \frac{\mathrm{Pe}_0}{\mathrm{Pe}_{s1}} \frac{h_2}{d_1} \frac{d\theta_{s1}}{d\xi} ; \tag{38}$$

$$\xi = 1 : \quad \frac{d\theta_{f2}}{d\xi} = \frac{d\theta_{s2}}{d\xi} = 0 . \tag{39}$$

The conjugation conditions ($\xi = \xi_1$) are

$$\Delta p' = p'_2 - p'_1 = \hat{J}^* \left(\frac{1}{\varepsilon_1 \rho'_{f1}} - \frac{1}{\varepsilon_2 \rho'_{f2}}\right), \tag{40}$$

$$\Delta\theta_f = \theta_{f2} - \theta_{f1} = \frac{p_{atm}}{p_0} \left(\frac{p'_2\left(\frac{p_0}{p_{atm}} - 1\right) + 1}{\rho'_{f2}} - \frac{p'_1\left(\frac{p_0}{p_{atm}} - 1\right) + 1}{\rho'_{f1}} \right), \tag{41}$$

$$\frac{1}{\mathrm{Pe}_{f2}} \frac{d\theta_{f2}}{d\xi} - \frac{1}{\mathrm{Pe}_{f1}} \frac{d\theta_{f1}}{d\xi} = \Delta\theta_f , \tag{42}$$

$$\theta_{s1} = \theta_{s2} , \quad \frac{d\theta_{s1}}{d\xi} = \Lambda \frac{d\theta_{s2}}{d\xi} . \tag{43}$$

Subject to relations (12), the conductive components in heat-transfer equations (34) and (35) are

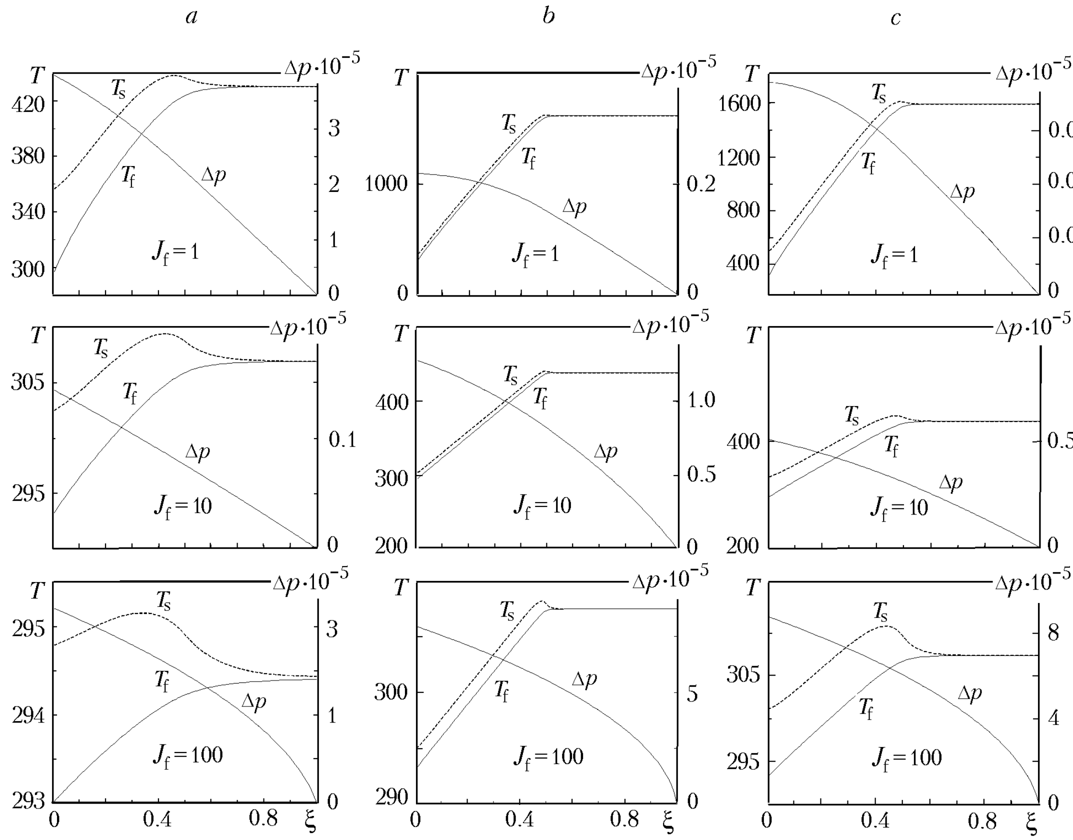

Fig. 3. Dependence of T_s, T_f, and Δp on the dimensionless coordinate ξ (Q = $5\cdot 10^7$ W/m^3): a) h_2 = 0.01 m, d = 0.001 m; b) 0.1 and 0.001; c) 0.1 and 0.003.

$$\frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_{fi}}\frac{d\theta_{fi}}{d\xi}\right)=\frac{1}{\mathrm{Pe}_{fi}}\frac{d^2\theta_{fi}}{d\xi^2}+\left(\frac{d\theta_{fi}}{d\xi}\right)^2\frac{d}{d\theta_{fi}}\left(\frac{1}{\mathrm{Pe}_{fi}}\right)=\frac{1}{\mathrm{Pe}_{fi}}\frac{d^2\theta_{fi}}{d\xi^2}+\left(\frac{d\theta_{fi}}{d\xi}\right)^2\frac{T_0}{\mathrm{Pe}_{fi}}\left(\frac{0.000145}{\lambda_{fi}(T_{fi})\,T_{fi}^{0.1557}}\right), \tag{44}$$

$$\frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_{si}}\frac{d\theta_{si}}{d\xi}\right)=\frac{1}{\mathrm{Pe}_{si}}\frac{d^2\theta_{si}}{d\xi^2}+\left(\frac{d\theta_{si}}{d\xi}\right)^2\frac{d}{d\theta_{si}}\left(\frac{1}{\mathrm{Pe}_{si}}\right)=\frac{1}{\mathrm{Pe}_{si}}\frac{d^2\theta_{si}}{d\xi^2}+$$

$$+\left(\frac{d\theta_{si}}{d\xi}\right)^2\frac{T_0}{\lambda_{si}(T_{si})\,\mathrm{Pe}_{si}}\left(0.8443\cdot\frac{\lambda_{c-ci}(T_{si})}{T_{si}}-\frac{9.5167\cdot 10^{-5}\,\mathrm{Re}_i(T_{si})}{T_{si}^{0.1557}}+6.2325\cdot 10^{-10}T_{si}^2\right). \tag{45}$$

We note that, according to the recommendations given in [12], the coefficient of interphase heat transfer defined by Eqs. (10) and (11) is calculated at a temperature $(T_{si}+T_{fi})/2$. With allowance for the Reynolds analogy [13], the viscosities μ_{fi} entering into Re_i (Eq. (33)) are also calculated at the same temperature.

Analysis of the Results Obtained. Figures 3–5 present the dependences of T_s, T_f, and $\Delta p = p - p_{atm}$ on the coordinate x (ξ) that were obtained as a result of solving Eqs. (33)–(36) with the corresponding boundary conditions for different mass fluxes of heat carriers, diameters of particles, and heights of granular beds (in all cases $\varepsilon_1 = \varepsilon_2 = \varepsilon = 0.4$; $d_1 = d_2$; $h_1 = h_2/2$). The release of heat in the active zone leads to a substantial dependence of the temperatures of the gas and particles on the characteristics of this zone Q and h_1. The value of the gas temperature at the exit from the system is easily determined from the balance relation

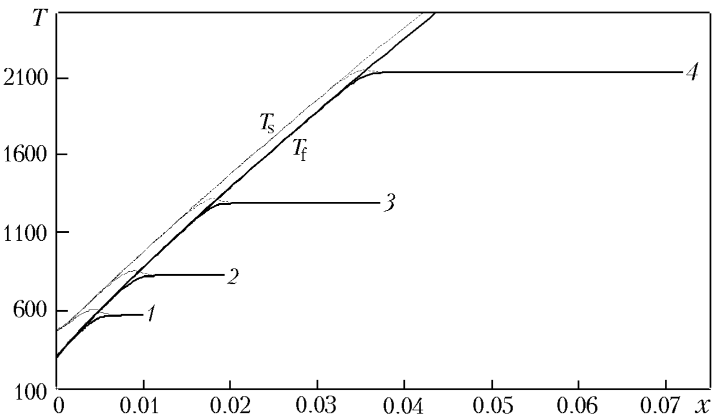

Fig. 4. Dependence of T_s and T_f on the coordinate x (Q = $5{\cdot}10^3$ W/m^3; d_1 = d_2 = $1{\cdot}10^{-3}$ m, J_f = 1 kg/(m$^2{\cdot}$sec)): 1, 2, 3, and 4) h_2 = 0.01, 0.0193, 0.0373, and 0.072, respectively.

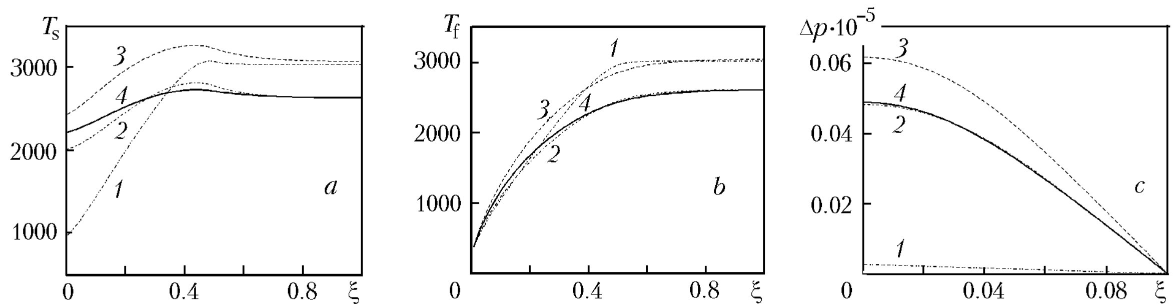

Fig. 5. Comparison between the solutions of different approximations of system (1)–(4) (Q = $5{\cdot}10^9$ W/m^3, d_1 = d_2 = $1{\cdot}10^{-3}$ m, h_2 = 0.01 m, J_f = 1 kg/(m$^2{\cdot}$sec)): 1) zero approximation (λ_s and λ_f, constants at T_0); 2) first approximation (conductive terms in (2) and (3); i.e., $\varepsilon\lambda_f(T_f)\frac{d^2T_f}{dx^2}$ and $(1-\varepsilon)\lambda_s(T_s)\frac{d^2T_s}{dx^2}$ without allowance for radiation); 3) second approximation (conductive terms in (2) and (3); i.e., $\frac{d}{dx}\left(\varepsilon\lambda_f(T_f)\frac{dT_f}{dx}\right)$ and $\frac{d}{dx}\left((1-\varepsilon)\lambda_s(T_s)\frac{dT_s}{dx}\right)$ without allowance for radiation); 4) "exact" solution of system (1)–(4).

$$c_pJ_f\,(T_{f2}\,(h_2)-T_0)=Q\,(1-\varepsilon_1)\,h_1\,, \tag{46}$$

which for the dimensionless quantity θ_{f2} (1) yields the simple equation

$$\theta_{f2}\,(1)=\hat{Q}\,\frac{h_1}{h_2}\,. \tag{47}$$

In practice, of great value is estimation of the maximum temperature of particles which is attained in the region $x\approx h_1$. To calculate this value, the following approximation was obtained:

$$\theta_{s,\max}-\theta_{f2}\,(1)=0.31\hat{Q}^{\,0.16}\,\hat{Q}_2^{\,0.43}\,, \tag{48}$$

which takes into account the influence exerted on $\theta_{s,max}$ by the main factors Q, d, h_1, and J_f. The pressure drop in a granular bed may attain great values. The calculated dependence has the form

$$D_0 = \left(0.022\varepsilon^{-5.7}\,\mathrm{Re}_0^{1.55} + 0.006\varepsilon^{-1.85}\,\mathrm{Re}_0^{2.3}\right)\hat{Q}^{\,0.42}\,. \tag{49}$$

We note that by its structure Eq. (49) resembles Eq. (33). The influence of heat release (temperature factor) leads to a certain transformation of exponents at Re_0. We will analyze the active zone-average relative difference in the temperatures of phases $\eta = \langle 2(T_s - T_f)/(T_s + T_f)\rangle$. To calculate this value, the following expression was obtained:

$$\eta = 0.25\hat{Q}_2^{\,0.73}\,. \tag{50}$$

It is evident that the parameter η characterizes the applicability of the two-temperature model and the complex $\hat{Q}_2$ makes it possible to determine the fields of applicability of the one- and two-temperature models. If we take $\eta = 0.01$, then it is seen from Eq. (50) that for $\hat{Q}_2 < 0.01$ the one-temperature model can be used:

$$c_p\,\rho_f\varepsilon v_f\frac{dT}{dx} = \frac{d}{dx}\left(\lambda_{\mathrm{eff}}\frac{dT}{dx}\right) + Q\,(1-\varepsilon)\,, \tag{51}$$

which follows from (2) and (3) at $T_s \approx T_f$. The effective thermal-conductivity coefficient of the blown-through dispersed medium is calculated from the formula

$$\lambda_{\mathrm{eff}} = \varepsilon\lambda_f + (1-\varepsilon)\,\lambda_s\,. \tag{52}$$

When $\hat{Q}_2 > 0.01$, it is necessary to take into account the difference between the temperatures of phases and apply Eqs. (2) and (3).

Figure 4 shows the behavior of temperatures T_s and T_f at different heights of a granular bed. As is seen, the values of the temperatures of phases for beds with different h_2 are strictly coordinated and form a single set. Figure 5 presents the results of comparison between the solutions of different approximations of system (1)–(4) that reflect the influence of radiation and of the nonlinearity of Eqs. (2) and (3) on the temperatures of phases and pressure. As is seen, T_s, T_f, and p found as a result of the "exact" solution of system (1)–(4) agree satisfactorily with the values obtained from an approximate solution (the conductive terms in Eqs. (2) and (3) are represented as $\varepsilon\lambda_f\frac{d^2T_f}{dx^2}$ and $(1-\varepsilon)\lambda_s(T_s)\frac{d^2T_s}{dx^2}$ without allowance for radiation). It is difficult to explain this result qualitatively because of the complexity of Eqs. (1)–(4). Nevertheless, this is a certain justification of the practical use of the simplified heat-conduction equations of phases with variable coefficients λ_f and λ_s without allowance for radiation heat transfer.

In conclusion, we note that approximating dependences (48), (49), and (50) were obtained for the following ranges of dimensionless parameters: $27.7 \le \mathrm{Re}_0 \le 16{,}610$, $0.0013 \le \hat{Q} \le 9.94$, $3.3\cdot10^{-7} \le \hat{Q}_2 \le 0.09$, and $59 \le D_0 \le 8.7\cdot10^7$.

NOTATION

c_p and c_v, specific heat capacities of gas at constant volume and pressure, respectively, J/(kg·K); d and d_i, diameters of particles, m; $D_i = ((p_0 - p_{\mathrm{atm}})/h_2)d_i^3\rho_{fi}/\mu_{fi}^2$; $D_0 = ((p_0 - p_{\mathrm{atm}})/h_2)d_1^3\rho_{f0}/\mu_{f0}^2$; h_1, height of the active (heat-generating) bed, m; h_2, total height of a granular bed, m; $J_f = \rho_{fi}\varepsilon_i v_{fi}$, mass flow rate of gas, kg/(m^2·sec); $\hat{J}_i = J_f^2 d_i^3/(\varepsilon_i h_2\mu_{fi}^2)$; $\hat{J}^* = J_f^2/(p_0 - p_{\mathrm{atm}})\rho_{f0}$; $\mathrm{Pe}_0 = c_pJ_fd_1/6\alpha_0h_2(1-\varepsilon_1)$, $\mathrm{Pe}_i = c_pJ_fd_i/(6\alpha_ih_2(1-\varepsilon_i))$, $\mathrm{Pe}_{fi} = c_pJ_fh_2/(\varepsilon_i\lambda_{fi})$, $\mathrm{Pe}_{si} = c_pJ_fh_2/[(1-\varepsilon_i)\lambda_{si}]$, Péclet numbers; Pr, Prandtl number; p_i, pressure, Pa; p_0, pressure at inlet into a granular bed, Pa;

$\Delta p = p - p_{atm}$; $p'_i = (p_i - p_{atm})/(p_0 - p_{atm})$; q, q_0, and q_1, heat fluxes, W/m^2; Q, heat-generation power, W/m^3; $\hat{Q} = Q(1 - \varepsilon_1)h_2/(c_p J_f T_0)$; $\hat{Q}_2 = Q(1 - \varepsilon_1)d/(c_p J_f T_0)$; $\mathrm{Re}_i = J_f d_i/\mu_{fi}$ and $\mathrm{Re}_0 = J_f d_1/\mu_{f0}$, Reynolds numbers; R, gas constant, m^2/(sec^2·K); S, cross section of a bed, m^2; S_{in}, specific surface of particles (interphase surface per unit volume of bed) in the case of a packing of spheres $S_{in} = 6(1 - \varepsilon)/d$, m^2/m^3; T_{fi} and T_{si}, temperature of gas and particles, K; T_0, inlet gas temperature, K; T'_0, gas temperature for $x \to -0$, K; v_{fi}, gas velocity in the interstices between particles, m/sec; u_{fi}, gas-filtration velocity, m/sec; x, coordinate, m; α_0, heat-transfer coefficient, W/(m^2·K); $\alpha_0 = 0.5 c_p J_f \mathrm{Re}^{-0.5}\mathrm{Pr}^{-0.6}$ [9], W/(m^2·K); α, coefficient of interphase heat transfer, W/(m^2·K); δ_{1i}, Kronecker symbol; ε, porosity; $\theta_{fi} = (T_{fi} - T_0)/T_0$ and $\theta_{si} = (T_{si} - T_0)/T_0$, dimensionless temperatures of gas and particles; κ, absorption coefficient of dispersed medium, 1/m; λ_f^0, molecular thermal conductivity of gas, W/(m·K); $\Lambda = \frac{(1 - \varepsilon_2)\lambda_{s2}}{(1 - \varepsilon_1)\lambda_{s1}}$; λ_{fi} and λ_{si}, effective thermal conductivities of gas and particles, W/(m·K); λ_e, eddy heat-conduction coefficient, W/(m·K); μ_{fi}, dynamic viscosity of gas, kg/(m·sec); μ_{f0}, dynamic viscosity of gas at atmospheric pressure and temperature T_0, kg/(m·sec); $\xi = x/h_2$, $\xi_1 = h_1/h_2$; ρ_f, gas density, kg/m^3; $\rho'_{fi} = \rho_{fi}/\rho_{f0}$; ρ_{f0}, gas density at atmospheric pressure and temperature T_0, kg/m^3; σ, scattering coefficient of dispersed medium, 1/m. Superscript: 0, molecular. Subscripts: atm, atmospheric; c-c, conductive-convective; e, eddy; eff, effective; f, fluid (gas); in, interphase; i, number of a granular bed; max, maximum; 0, at the inlet; p, at constant pressure; r, radiant; s, solid particles; v, at constant volume.

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